An optimization model for the outbound truck scheduling problem at cross-docking platforms

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Abstract A cross-dock is a facility where arriving materials are sorted, grouped and delivered to destinations, with very limited storage times, with the overall objective of optimizing the total management costs. The operational efficiency of a cross-docking system strongly depends on how the logistic activities are organized. For this reason, optimization models and methods can be very useful to improve the system performances. In this paper, we propose a mathematical model to describe the so-called truck scheduling problem at a cross-docking platform. The model considers most of the actual constraints occurring in real problems; therefore, it can be viewed as an interesting basis to define a decision support system for this kind of problems. Some preliminary results show that the model can be efficiently solved in limited computational times.

Keywords: supply chain, cross-docking, truck scheduling

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Introduction

A cross-dock is a facility that receives goods from suppliers and sorts them into alternative groups, based on the downstream delivery points. This way, it is possible to reduce the total distribution costs taking advantage of the benefits of a warehousing strategy in terms of consolidation (enabling economies of scale through the consolidation of multiple less-than-truckload shipments), but avoiding storage costs and reducing handling operations.

In the literature, several models and methods have been proposed to address different kinds of cross-docking optimization problems. In this paper, we propose a mathematical model to describe the so-called truck scheduling problem at a cross-dock.

The model considers most of the actual constraints occurring in real-world problems, striving to define a decision support system for the management of cross-docking platforms.
The remainder of the paper is arranged as follows. In the next section, a review of the extant literature dealing with cross-docking optimization highlights the main research strands and potential gaps. Then, the description of the model is provided. Computational results are then illustrated, showing that the model can be efficiently solved in limited computational times.

Literature Review

In recent years, many attempts have been made in order to systematize the cross-docking literature [1-5]. In particular, Buijs et al. [5] classified problems on the basis of spatial and temporal aspects, distinguishing between single cross-dock and cross-docking network management problems, and, according to the temporal dimension, among strategic, medium term and operational problems. Most of the literature concerns with operational decision-making problems at a local level; in particular with:

- the inbound truck scheduling problem, consisting in the assignment of the inbound trucks to the receiving doors and of their subsequent scheduling;
- the outbound truck scheduling problem, arising when, starting from the inbound trucks' arrival scheduling, the loading and the scheduling of outbound vehicles should be determined, along with their assignment to shipping doors;
- the synchronization truck scheduling problem, arising when both the previous problems have to be simultaneously solved.

Considering the interdependencies among the above problems, global optimization approaches should include all the planning and management aspects. However, the relevant complexity of the single sub-problems suggests the development of separated models and methods.
Most of the papers in the literature consider very simplifying assumptions, representing the cross-docking facility with one receiving door, one shipping door and an infinite staging area capacity [6-10]. With a slight variation, Vahdani and Zandieh [11] and Soltani and Saldjadi [12] considered a cross-dock that does not allow storage. Chen and Lee [13] solved the one inbound – one outbound door truck scheduling, modelling it as a detailed scheduling problem. Alpan et al. [14] dealt with a multi-door cross docking problem, considering temporary and limited storage. Boysen et al. [15-16] determined the schedule sequences for inbound trailers in a multiple door cross-dock. Similar problems were also tackled by Liao et al. [17], who also dealt with inbound truck assignment to dock doors and outbound trucks. Konur and Golias [18] introduced uncertainty in inbound truck arrival times, assuming just arrival time windows are known.
Miao et al. [19] were the first to consider a synchronization problem in a multiple doors cross-dock. Chen and Song [20] extended the work of Chen and Lee [13], to solve the problem with multiple doors for inbound and outbound processes. The

integration of Vehicle Routing Problem into a cross-docking system was also considered [21-23].

Most of the proposed cross-docking scheduling problems deal with fixed outbound schedules and just optimise inbound truck processing. Indeed, Boysen et al. [16] mention that fixed outbound schedule represents an important real-world aspect of cross docking. This is true especially in large hub-and-spoke networks. However, in industries characterised by less-than-truckload logistics and specific deadlines for goods (such as postal services or food supply chains), where firms mainly transport comparatively small and low-valued shipments of multiple senders, things may change quite significantly. Indeed, in this kind of industry the need for consolidation, load optimization and minimization of number of outbound trucks, while respecting deadlines, is even more important, due to tight profit margins. For this reason, this paper will propose a mathematical programming framework to deal with outbound truck scheduling at cross-docking platforms. The model assumes that inbound track sequencing is known a-priori, and seeks to minimize the number of departures towards a set of destinations. The proposal may be viewed as an extension of models introduced by Bruno et al. [24-25] for the bus scheduling at a transit terminal. In the next section, the model will be described firstly considering the basic case of one inbound – one outbound door, and then extended to the general case with more inbound and outbound doors.

A general framework model for cross-docking truck scheduling

One inbound - one outbound door case

For sake of clarity, we initially describe the proposed model with reference to the outbound truck scheduling problem for the one inbound – one outbound door case (see also Bruno et al. [26]).

In particular, dividing the time horizon into n time periods, the time expansion of the terminal over the time horizon $[0,T]$, is given by a graph of n nodes, in which each node t corresponds to a copy of the terminal in the time t, that is linked with the node $t+1$ by an holdover arc (Figure 1a).

At a given time, an inbound truck delivers a certain number of lots at the door of the cross-dock. Each lot is generally characterized by a destination o and a deadline d, within which it needs to leave the dock. Sets of lots with the same destination can be grouped together and leave the cross-dock in t, if there is an outbound truck leaving toward that destination in t. Otherwise, they can be stored in the terminal, flowing through the holdover arcs.

Assuming as parameters:

- O, set of the possible destinations ($o \in O$);
- D, set of the possible deadlines ($d \in D$);
- d_{td}^{o}.lots with deadline d and destination o arrived at the cross-dock at time t;
- f_t, cost associated with the vehicle leaving the cross-dock at time t;

- Q_t, capacity of the outbound vehicles, i.e. the maximum number of lots which can be loaded on an outbound truck in period t;
- C, capacity of the cross-dock, i.e. the maximum number of lots that can be stored at the facility (assumed to be constant over the time);

and introducing as decision variables:

- x_{td}^o, number of lots with deadline d and destination o stored in $[t, t+1]$;
- q_{td}^o, number of lots with deadline d and destination o leaving at time t;
- y_t^o , binary variable equal to 1 if a truck leaves the cross-dock at time t toward the destination o, 0 otherwise;

the formulation of the model is given by:

(1) $$\min z = \sum_{t=1}^{n} \sum_{o \in O} f_t \, y_t^o$$

(2) $x_{td}^o = x_{t-1,d}^o + d_{td}^o - q_{td}^o$ $\quad \forall t = 1, .., n; \forall o \in O; \forall d \in D$

(3) $\sum_{d \geq t} q_{td}^o \leq Q_t \, y_t^o$ $\quad \forall t = 1, .., n; \quad \forall o \in O$

(4) $\sum_{o \in O} \sum_{d \in D} x_{td}^o \leq C$ $\quad \forall t = 1, .., n$

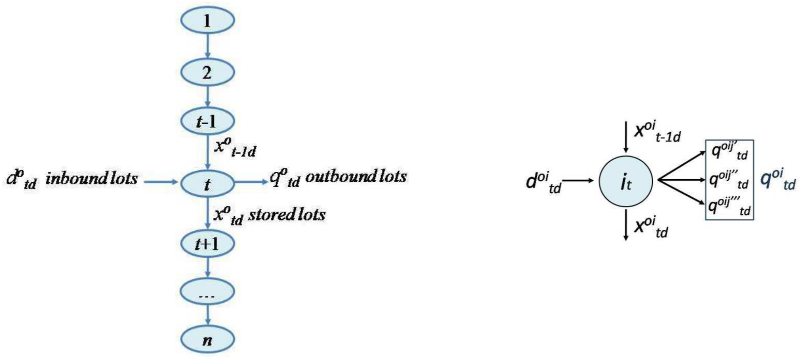
(5) $\sum_{o \in O} y_t^o \leq 1$ $\quad \forall t = 1, .., n$

(6) $x_{0d}^o = x_{dd}^o = 0$ $\quad \forall o \in O; \forall d \in D$

(7) $x_{td}^o, q_{td}^o \geq 0$ $\quad \forall t = 1, .., n; \forall o \in O; \forall d \in D$

(8) $y_t^o \in \{0; 1\}$ $\quad \forall t = 1, .., n; \forall o \in O$

The objective function (1) is the sum of the costs associated with the activation of outbound trucks across the planning horizon. Constraints (2) are the so-called mass balance constraints, i.e. the conditions about the flow material balance at each time t. Conditions (3) and (4) assure that capacity constraints of the cross-dock and of the outbound trucks are satisfied in each time period. Constraints (5) indicate that no more than one truck can leave the cross-dock at each time t, because of the presence of a single shipping door; constraints (6) are related to the deadlines as they assure that no lot stay inside the cross-dock after their own deadlines. Constraints (7-8) define the nature of the introduced binary variables.



(a) single door **(b) multiple doors**

Fig. 1 –Cross Docking Dynamic Network.

The multiple doors case

In this case, more than one truck can arrive at and depart from the terminal at each time t from different doors. Then, two further indices have been introduced to identify the receiving door i and the shipping door j respectively. The lots arriving at a given receiving door i (d_{td}^{oi}) can be temporary stored or directly moved towards one of the shipping doors j for departure. Lots are stored at the related receiving doors and picked up whenever they have to be loaded on a truck at a given door j. The times to transfer lots from receiving to shipping doors cannot be neglected; in particular, they have been assumed dependent on the specific pair (i, j).
Considering the following further notation:

- I, set of the inbound doors ($i \in I$);
- J, set of the outbound doors ($j \in J$);
- m_{ij}, time to transfer lots from the receiving door i to the shipping door j;

and introducing as decision variables (Fig. 1b):

- x_{td}^{oi}, number of lots coming from the receiving door i, with deadline d and destination o stored during the time interval $[t, t+1]$;
- q_{td}^{oij}, number of lots coming from the receiving door i with deadline d and destination o leaving at time t from the shipping doors j;
- y_t^{oj}, binary variable equal to 1 if a truck leaves from the shipping door j at time t to the destination o, 0 otherwise.

the model can be formulated as follows:

(9) $$\min z = \sum_{j\in J}\sum_{t=1}^{n}\sum_{o\in O} f_t y_t^{oj}$$

(10) $$x_{td}^{oi} = x_{(t-1)d}^{oi} + d_{td}^{oi} - \sum_{j\in J:(t+m_{ij})\le n} q_{(t+m_{ij})d}^{oij} \qquad \forall t = 1..n; o \in O; i \in I; d \in D$$

(11) $$\sum_{o\in O}\sum_{i\in I}\sum_{d\in D} x_{td}^{oi} \le C \qquad \forall t = 1,..,n$$

(12) $$\sum_{d\in D} q_{td}^{oj} \le Q_t y_t^{oj} \qquad \forall t = 1..n; o \in O; j \in J$$

(13) $$\sum_{o\in O} y_t^{oj} \le 1 \qquad \forall t = 1..n; j \in J$$

(14) $$x_{0d}^{oi} = 0 \qquad \forall o \in O; i \in I; d \in D$$

(15) $$\sum_{j\in J}\left[\sum_{t=1}^{d-m_{ij}} q_{td}^{oij}\right] = \sum_{t=1}^{d} d_{td}^{oi} \qquad \forall o \in O; i \in I; d \in D$$

(16) $$x_{td}^{oi} \ge 0 \qquad \forall t = 1..n; o \in O; i \in I; d \in D$$

(17) $$q_{td}^{oij} \ge 0 \qquad \forall t = 1..n; o \in O; i \in I; d \in D$$

(18) $$y_t^{oj} \in \{0; 1\} \qquad \forall t = 1..n; o \in O; j \in J$$

The objective function (9) is defined as the sum of the costs associated with the departure of outbound trucks during the planning horizon. Constraints (10) indicate the flow material balance at each receiving door i at each t. Conditions (11) assure that, in each period t, the lots stored at all the receiving doors i do not exceed the total capacity C of the dock. Conditions (12) guarantee that if in t a truck leaves from a generic shipping door j toward the destination o (y_t^{oj} =1), only the lots with the same destination can be loaded on the truck, without exceeding its capacity. Constraints (13) indicate that no more than one truck can depart from the shipping door j at each time t. Constraints (14) impose that the stock level at each receiving door i is zero at the beginning of the planning horizon; while (15) assure that no lot stay inside the cross-dock after their own deadlines. Conditions (16-18) define the nature of the introduced decision variables. It has to be noticed that conditions (15) can be also alternatively formulated as follows:

$$(15.1) \quad q_{td}^{oij} = 0 \qquad \forall i \in I; o \in O; j \in J; d \in D; t = (d+1)..n$$

$$(15.2) \quad x_{td}^{oi} = 0 \qquad \forall i \in I; \forall o \in O; \forall d \in D; \forall t = \left(d - \min_{j \in J} m_{ij} + 1\right)..n$$

Constraints (15.1) impose that lots with deadline d cannot be loaded on a truck later than d, at any shipping door j. Conditions (15.2) indicate the last period in which lots with deadline d may be stored at each receiving door i. In particular, for each i, there exists a latest period in which lots with deadline d can be picked up and moved toward the shipping door j ($t_{ij}^d = d - m_{ij}$). After that, they may be transferred only towards those doors j' reachable from i in less than m_{ij}.Then, the latest period in which lots with deadline d may be picked up from i is $t_i^d = d - \min_{j \in J} m_{ij}$.

Computational Experiences

In order to test the suitability of the proposed model, a set of instances was produced using a random generator designed and implemented in C++ language. ILOG CPLEX Optimization Tool was used to solve the randomly generated instances for the case of unlimited dock capacity, by varying the number of time periods T (from 12 to 36), the number of destinations O (from 2 to 6) and the number of inbound and outbound doors |I|=|J| (from 2 to 6). Results, reported in Table 1, show that computational times grow in a reasonable way in all cases. Even in the case of 36 times periods, 6 destinations and 12 doors (6 inbound and 6 outbound doors,) the solver is capable of finding a solution to the problem within four minutes.

Conclusions

In this paper, we analyzed the cross-docking approach as a tool to improve the performance of a delivery system within a supply chain context. Literature on this topic has shown that efficiency and effectiveness of cross-docking strategies strongly depend on the availability of optimization models and algorithms able to support the decision maker about the choices on the operational aspects. However, current literature on the truck scheduling just provides models in order to solve specific versions of the problem. For this reason, a general framework has been introduced, based on a mathematical model able to describe most of the scenarios, which can occur in practical applications. The first provided results show that the model produces interesting results both in term of computational efficiency and from a managerial point of view. Further investigations will be aimed at improving the computational efficiency of the model, through purpose-built solution methodologies.

T=12									
|I|=|J|	**|O|=2**			**|O|=4**			**|O|=6**		
	Avg	Min	Max	Avg	Min	Max	Avg	Min	Max
2	0.30	0.18	0.38	0.47	0.38	0.53	0.63	0.35	0.91
4	0.31	0.29	0.32	1.50	0.66	2.10	18.00	1.70	75.00
6	0.64	0.28	1.46	1.53	0.64	2.42	32.00	1.30	145.00
T=18									
|I|=|J|	**|O|=2**			**|O|=4**			**|O|=6**		
	Avg	Min	Max	Avg	Min	Max	Avg	Min	Max
2	0.34	0.25	0.45	0.64	0.39	0.98	0.71	0.39	0.98
4	0.61	0.37	0.84	9.10	0.70	24.00	28.00	21.00	40.00
6	1.80	0.62	3.26	10.30	3.90	27.00	146.00	68.00	214.00
T=24									
|I|=|J|	**|O|=2**			**|O|=4**			**|O|=6**		
	Avg	Min	Max	Avg	Min	Max	Avg	Min	Max
2	0.70	0.37	1.28	0.82	0.57	1.32	1.20	0.79	1.61
4	0.79	0.58	0.94	9.50	1.90	31.00	52.00	7.50	121.00
6	2.40	1.00	3.40	51.00	22.00	82.00	158.00	71.00	220.00
T=36									
|I|=|J|	**|O|=2**			**|O|=4**			**|O|=6**		
	Avg	Min	Max	Avg	Min	Max	Avg	Min	Max
2	1.70	0.64	3.09	2.60	1.20	3.60	3.40	1.30	5.70
4	1.90	1.00	3.10	16.40	4.30	18.70	56.00	13.00	58.00
6	11.30	6.50	15.30	66.00	18.70	73.00	241.00	93.00	265.00

Table1 – Run Times (seconds)

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